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BOOK REVIEW

Introduction to Vortex Theory. By H. J. Lugt. Vortex Flow Press, Potomac, MD, USA, 1996. 627 pp. ISBN 0965 768902.

This book presents a broad review of vortex dynamics in incompressible flows. It is aimed at a graduate-level audience as well as researchers seeking an overview of a highly developed, and still developing subject. While a diverse range of topics are covered, the book falls well short of a uniformly comprehensive review of the subject. Some topics, such as distributed vortices in two-dimensional flows, high-Reynolds-number turbulence and the role of stratification are treated in a cursory manner, while others, such as vorticity generation at a solid boundary (turbines, wings, etc.), highly singular vortex sheet behaviour and low-Reynolds-number vortex–body interactions are strongly emphasized. This style reflects the author's research interests, and students and researchers who share those interests would certainly benefit from this work. Its wider relevance say to students and researchers in atmospheric and oceanic dynamics, where vortex theory plays a crucial role, is questionable.

The author begins by attempting to define a vortex, and he returns to this problem several times throughout the remainder of the book. To develop a 'vortex theory', as the title advertises, it would appear necessary to define a vortex, but in fact this goal is not fully achieved. The author points out the ambiguity of using various simple definitions – like a local region of anomalously high vorticity – and suggests that, in general, a vortex cannot be rigorously defined, except in one case. That is the case of inviscid, two-dimensional flows, where he borrows the definition introduced by Saffman & Baker (*Ann. Rev. Fluid Mech.* vol. 11, 1979): 'A vortex in an inviscid fluid is a finite, simply-connected region of vorticity, surrounded by irrotational motion or boundaries'. But this definition itself is highly problematic, for one of the properties of vorticity in two-dimensional inviscid flows, where vorticity is a materially conserved quantity, is that it tends to develop highly intricate structure in all but the simplest flows, i.e. in flows where the vortex does not change shape (cf. Dritschel, *Computer Phys. Rep.* vol. 10, 1989, p. 77, and references therein). In unsteady situations, it rapidly becomes futile to distinguish a simply connected and a multiply connected region of vorticity. For a vortex surrounded by a sea of fine-scale filamentary vorticity, what part of this sea should be considered part of the vortex? Does it make sense to include filamentary vorticity being stripped away from a vortex at an exponential rate? I would maintain that a pertinent, rigorous definition of a vortex is not even available for this case.

The book goes on to cover some background material, such as streamlines, basic conservation laws, and the Navier–Stokes equations. Then special solutions of the equations are described that introduce 'vortices'. This is followed by a discussion of the vorticity equation, boundary conditions, and circulation, and the following three chapters are devoted to singular distributions of vorticity – point vortices and vortex sheets – collected under the inappropriate heading 'Potential Flow'. Some errors are made in the discussion on the conserved quantities for point-vortex systems (there are four not three), and the conditions for integrability of the dynamical system. Vortex sheets are introduced without discussion of their well-known short-wave instability that renders them ill-posed for inviscid flows. That instability is described several chapters later, but it is used there only to explain the long-wavelength roll-up of the sheet, to connect it with experimental results for viscous shear layers. For the latter, and for any finite-width shear layer, the vorticity evolution is well posed – short wavelengths are stable. A truly inviscid vortex sheet can be made to mimic a predominant long-wavelength growth initially by careful smoothing of the initial sheet and delicate numerical methods that suppress noise growth. Nature is not likely to be as careful as the computer programmer, however. Any background noise having a power-law spectrum at short wavelengths will lead to rapid crinkling at the smallest scales first, simply because disturbances grow exponentially at a rate inversely proportional to wavelength. This simple observation soils the mathematical beauty that attracted so much interest in vortex sheets. Why this model has received such interest is beyond the scope of this review.

Non-uniform two-dimensional and three-dimensional inviscid vortices are considered next. Here some of the remarks about two-dimensional vortices are misleading, such as calling the vorticity discontinuity at the edge of a patch of uniform vorticity a ‘singularity’, and the entire description of ‘contour surgery’ (Dritschel, *op. cit.*). No singular behaviour of any kind ever occurs: unlike a vortex sheet, a vorticity discontinuity is well behaved for all time, and may be considered as a basic building block for structures in inviscid two-dimensional flows. For three-dimensional vortices, the focus is on vortices of unchanging shape (e.g. uniformly propagating vortex rings), and on various models for slender vortex filaments. No discussion is presented of unsteady three-dimensional inviscid vortex motion anywhere in the text, apart from a few special cases presented in this chapter. For very high-Reynolds-number flows, multiple vortex filament numerical methods based on the Biot–Savart law have been used widely by Leonard, Meiburg and others. Overall, inviscid vortex theory is poorly developed in this book.

The next chapter discusses instability. More emphasis could have been given to stability, both linear and nonlinear conditions for. The derivation of the Kelvin–Helmholtz instability is given in a classical manner, whereas in a book on vortex theory, it would have been more appropriate to attack this problem directly from the vorticity equation, i.e. the Birkoff–Rott equation in this case. This would show directly that one need only make reference to the vorticity field and would further emphasize the fundamental role of vorticity in incompressible flows.

The next few chapters are devoted to viscous vortices. This is well developed and clearly reflects the author’s research interests. In fact, the author states at the beginning of chapter 12: ‘We have arrived at a point in our endeavor that will surely give us the most intellectual satisfaction when we simulate nature mathematically: the description of vortices in a real fluid by means of solutions to the Navier–Stokes equations...’. By ‘real’, the author indicates that he means ‘viscous’. This statement, in my opinion, shows the author’s lack of appreciation for the very real flows in the atmosphere and oceans, subjects briefly touched upon at the end of the book. Such flows are indeed viscous, but the Reynolds numbers are so large for phenomena of interest that they can be regarded as perfectly inviscid. Dissipation does occur in these fluids, but there are other mechanisms like thermal damping that largely account for it. No weather forecasting model would ever use molecular viscosity; only for numerical stability purposes is an *ad hoc* coefficient of viscosity, orders of magnitude greater than molecular viscosity, often used.

Notwithstanding this, the author does go into considerable depth in describing vorticity generation from a solid and a free surface, and a variety of vortex–body interactions such as a vortex ring impinging obliquely on a wall. There is also a description of vortices moving in a viscous fluid far from boundaries, including the important reconnection process of two vortices in a three-dimensional flow. Much detail is given for flow around obstacles (idealized airfoils), but this is primarily at low to moderate Reynolds number.

A special chapter is devoted to 'swirling motion', though this can be regarded as arising from a three-dimensional vorticity field with curved (e.g. helical) vortex lines. The inviscid theory for steady axisymmetric flow is first developed, and then viscous modifications are discussed, en route to discussing the important phenomenon of vortex breakdown. Another special chapter follows on 'vortex sound'; the author here partially relaxes the incompressibility constraint by looking at linear acoustic wave generation from otherwise incompressible, unsteady vortex motion. Lighthill's classical theory is preceded by a somewhat mysterious 'black box' method, supposedly useful for modelling vortex-induced oscillations or rotations of bodies in fluids. Then follows a twenty-page chapter on turbulent vortices, which, in my opinion, is out of date. One could write an entire book on this subject, for instance including some of the numerical simulation results for three-dimensional Navier–Stokes turbulence that have been obtained by many researchers in recent years. It would have been instructive to review the characteristic forms taken by the dominant vortices in turbulence, i.e. sheet-like structures and tubes, as well as the observed link between the strain and the vorticity. In place of this, there is an antiquated discussion of 'eddy viscosity' going back to Boussinesq in the last century. The author admits, in the end, that 'the kaleidoscope of the many eddy-viscosity models is of no further concern here, since they do not really add much to our basic understanding of turbulent vortices.' Additional discussion is given for turbulent vortex rings, shear layers, wakes, and boundary layers.

The final two chapters discuss the effects of background rotation and of buoyancy, principally in connection with atmospheric and oceanic dynamics. A simple two-dimensional, barotropic model of the 'Rossby' waves arising in a shallow fluid layer (of uniform depth) from variations of the Earth's radial component of rotation is introduced without adequate motivation. The Taylor–Proudman theorem, discussed in the same section, presumably provides the motivation, but the connection is not made clear. In fact the connection is tenuous, for the effects of buoyancy, ever present in the atmosphere and oceans, lead to significant flow variations throughout the fluid depth, and hence oppose the effects of rotation. A clearer description of Rossby waves and other topics covered in this chapter might have resulted if buoyancy effects (discussed in the final chapter) were discussed before the effects of rotation. Then the effects of both buoyancy and rotation could be discussed in connection with atmospheric and oceanic dynamics, and the important principle of the material conservation of 'potential vorticity' could be introduced. This principle would allow a proper discussion of the competing effects of rotation and buoyancy, and indicate the conditions under which say rotational effects dominate and hence the Taylor–Proudman theorem applies. As it stands, these remaining topics are inadequately described. They could easily constitute a textbook themselves.

In summary, the scope of this book is too wide, and as a result various important topics have not been covered adequately. The best developed parts of this book concern viscous, incompressible vortex dynamics, a subject to which the author has contributed significantly. The recent growth of vortex dynamics outside of the traditional areas of aeronautics and hydrodynamics, in which vortex generation from and interaction with moving surfaces has been of principal concern, has created an exciting frontier in science. In atmospheric and oceanic dynamics, the vortex generation and dissipation mechanisms are very different, and molecular viscosity is utterly negligible at the scales of interest. Moreover, vortices move freely in a three-dimensional environment and may be conveniently identified by anomalous regions of 'potential vorticity'. The application of vortex dynamics in atmospheric and oceanic dynamics can now be regarded as a topic in its own right, and one that is sufficiently distinct from vortex dynamics in aeronautics and hydrodynamics to require an independent survey.

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